**Answer ALL TWENTY TWO questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

**1** Solve the simultaneous equations

$$① \quad x + 2y = 15 \quad \times 4$$

$$② \quad 4x - 6y = 4$$

Show clear algebraic working.

$$③ \quad 4x + 8y = 60$$
$$② \quad 4x - 6y = 4$$

$4x - 4x = 0$

$8y - -6y = 14y$

$$14y = 56$$
$$y = \frac{56}{14} = 4$$

sub into ①

$$x + 2y = 15$$
$$x + 8 = 15$$
$$x = 7$$

$x = $ 7

$y = $ 4

**(Total for Question 1 is 3 marks)**

**2** (*a*) Factorise $y^2 - 3y - 18$

1, 18
2, 9
3, 6 ~~4~~
~~6, 3~~

$-6 + 3 = -3$

$(y - 6)(y + 3)$ **(2)**

(*b*) Hence, solve $y^2 - 3y - 18 = 0$

$(y - 6)(y + 3) = 0$

$y - 6 = 0$ $\quad$ $y + 3 = 0$

$y = 6$ $\quad$ $y = -3$

$y = 6, y = -3$ **(1)**

**(Total for Question 2 is 3 marks)**

**3** On the grid, draw the graph of $5x + 2y = 10$ for values of $x$ from $-2$ to 4

$2y = -5x + 10$

$y = -2.5x + 5$

$x = 4$

$y = -2.5 \times 4 + 5$

$= -10 + 5$

$= -5$

$x = 0$

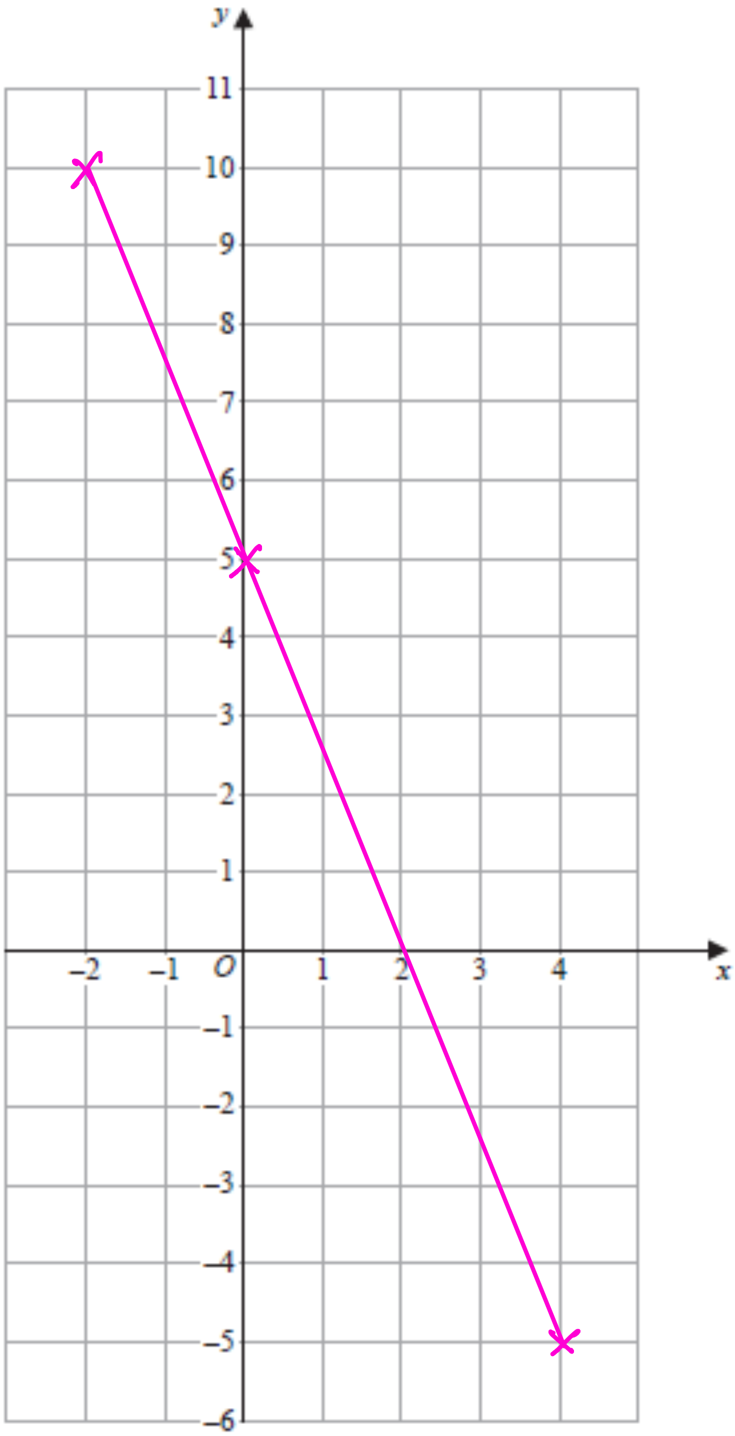
$y = 5$

$x = -2$

$y = -2.5 \times -2 + 5$

$= 5 + 5$

$= 10$




**(Total for Question 3 is 3 marks)**

**4** (*a*) Simplify $\frac{2}{y^0}$

$y^0 = 1$ so $\frac{2}{1} = 2$

2

**(1)**

(*b*) Simplify fully $\left(16a^4\right)^{\frac{3}{4}}$

$\left(\sqrt[4]{16}\right)^3$ $a^{4 \times \frac{3}{4}}$

$= 2^3 = 8$ $= a^3$

$8a^3$

**(2)**

**(Total for Question 4 is 3 marks)**

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**5** Factorise fully $18c^3d^2 - 21c^2$

1, 18
2, 9
3, 6
~~6, 3~~

1, 21
3, 7
~~7, 3~~

$3c^2(6cd^2 - 7)$

$3c^2(6cd^2 - 7)$

**(Total for Question 5 is 2 marks)**

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**6** Here is a list of six numbers written in order of size.

$$x \quad 5 \quad y \quad 8 \quad z \quad 10 \quad 12$$

9          10

The numbers have

a range of 9
a median of 8
a mode of 10

Find the value of $x$, the value of $y$ and the value of $z$

range of 9 so $12 - 9 = 3$ $x = 3$

mode of 10 so $z = 10$

median of 8 $y \; \overset{8}{\uparrow} \; 10$

so $y = 6$

$x =$ ..........3..........

$y =$ ..........6..........

$z =$ ..........10..........

**(Total for Question 6 is 3 marks)**

**7** Expand and simplify $5x(3x + 4)(2x - 1)$

$5x(6x^2 - 3x + 8x - 4)$

$= 5x(6x^2 + 5x - 4)$

$= 30x^3 + 25x^2 - 20x$

$30x^3 + 25x^2 - 20x$ ..................................................

**(Total for Question 7 is 3 marks)**

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**8** Solve $2^{-4x} = 32$

$32 = 2 \times 2 \times 2 \times 2 \times 2$

$= 2^5$

so $-4x = 5$

$x = -\frac{5}{4}$

$x =$ ....$-\frac{5}{4}$....

**(Total for Question 8 is 2 marks)**

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**9** (*a*) Write $9.32 \times 10^{-5}$ as an ordinary number.

$0.0000932$

**(1)**

(*b*) Work out $3 \times 10^5 - 6 \times 10^4$
Give your answer in standard form.

$3 \times 10^5$
$0.6 \times 10^5$
$2.4 \times 10^5$

$2.4 \times 10^5$

**(2)**

(*c*) Work out $(3 \times 10^{55}) \times (6 \times 10^{65})$
Give your answer in standard form.

$3 \times 6 \times 10^{55} \times 10^{65}$
$18 \times 10^{120}$

$1.8 \times 10^{121}$

**(2)**

**(Total for Question 9 is 5 marks)**

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**10** Show that $4\frac{2}{3} \div 1\frac{5}{6} = 2\frac{5}{11}$

$4\frac{2}{3} = \frac{14}{3}$

$1\frac{5}{6} = \frac{11}{6}$

$\frac{14}{3} \div \frac{11}{6}$

$= \frac{14}{\cancel{3}_1} \times \frac{\cancel{6}^2}{11}$

$= \frac{28}{11}$

$\frac{28}{11} = 2\frac{6}{11}$ as required.

**(Total for Question 10 is 3 marks)**

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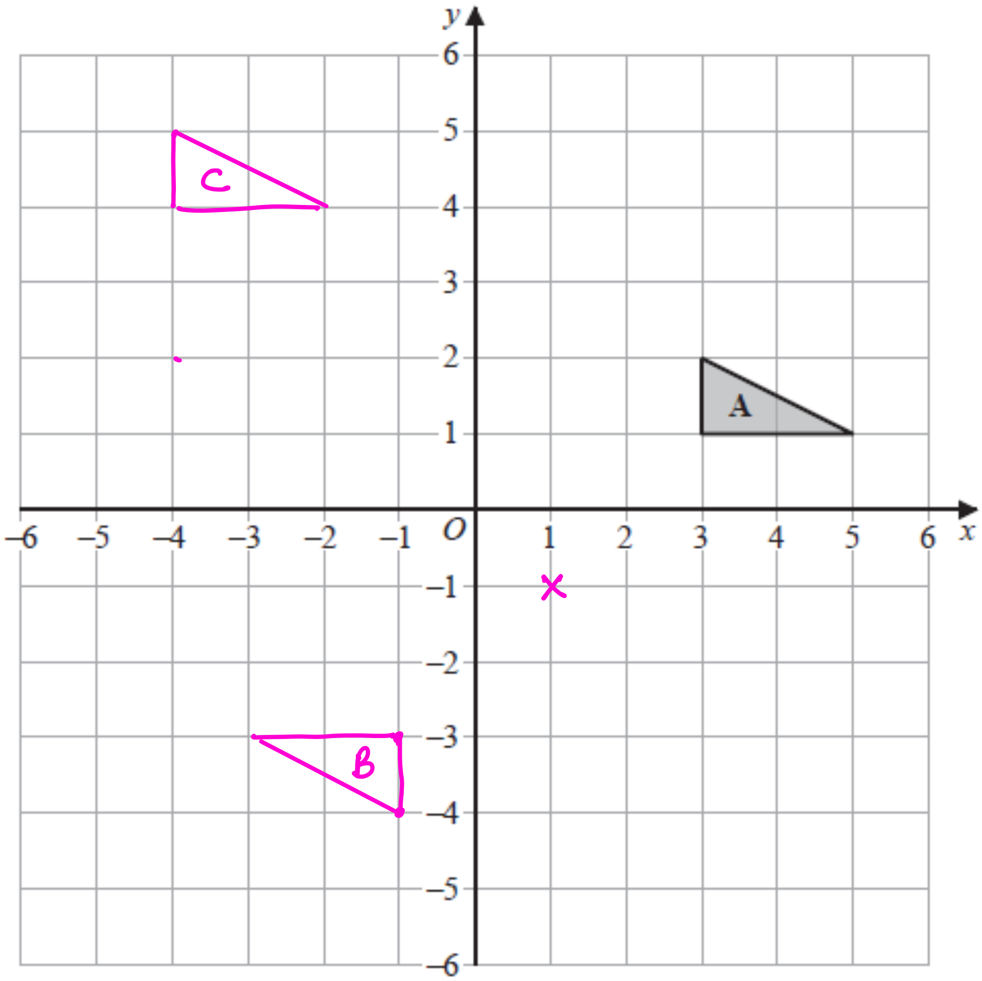
**11**




(*a*) On the grid, rotate triangle **A** 180° about (1, –1)
Label the new triangle **B**

**(2)**

(*b*) On the grid, translate triangle **A** by the vector $\begin{pmatrix} -7 \\ 3 \end{pmatrix}$

Label the new triangle **C**

**(1)**

**(Total for Question 11 is 3 marks)**

**12** The function f is such that

$$f(x) = \frac{2}{3x-5} \quad \text{where } x \neq \frac{5}{3}$$

(*a*) Find $f\left(\frac{1}{3}\right)$

$$\frac{2}{\cancel{3}\times\frac{1}{\cancel{3}}-5} = \frac{2}{-4} = -\frac{1}{2}$$

$-\frac{1}{2}$

**(1)**

(*b*) Find $f^{-1}(x)$

$$y = \frac{2}{3x-5}$$

$$3x - 5 = \frac{2}{y}$$

$$3x = \frac{2}{y} + 5$$

$$x = \frac{2}{3y} + \frac{5}{3}$$

$$f^{-1}(x) = \frac{2}{3x} + \frac{5}{3}$$

OR $= \frac{2+5x}{3x}$

$f^{-1}(x) =$ $\frac{2+5x}{3x}$

**(2)**

The function g is such that

$$g(x) = 5x^2 - 20x + 23$$

(*c*) Express g(*x*) in the form $a(x-b)^2 + c$

$$g(x) = 5(x^2 - 4x) + 23$$

$$= 5[(x-2)^2 - 4] + 23$$

$$= 5(x-2)^2 - 20 + 23$$

$$= 5(x-2)^2 + 3$$

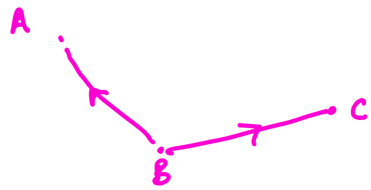
$5(x-2)^2 + 3$

**(3)**

**(Total for Question 12 is 6 marks)**

**13** Here are two vectors.

$$\overrightarrow{BA} = \begin{pmatrix} -5 \\ 4 \end{pmatrix} \qquad \overrightarrow{BC} = \begin{pmatrix} 9 \\ 1 \end{pmatrix}$$




Draw a sketch!

Find $\overrightarrow{AC}$ as a column vector.

$$\overrightarrow{AC} = -\overrightarrow{BA} + \overrightarrow{BC}$$

$$= \begin{pmatrix} 5 \\ -4 \end{pmatrix} + \begin{pmatrix} 9 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 14 \\ -3 \end{pmatrix}$$

**(Total for Question 13 is 2 marks)**

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**14** $-8 < 2y \leq 2$

$y$ is an integer.

(*a*) Find all the possible values of $y$

$$-8 < 2y \leq 2$$

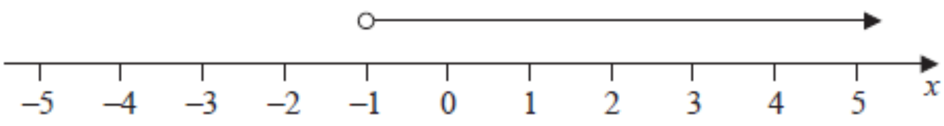
$$\div 2 \quad \div 2 \quad \div 2$$

$$-4 < y \leq 1$$

-3, -2, -1, 0, 1

**(2)**

(*b*) Write down the inequality shown on the number line.




$x > -1$

**(1)**

**(Total for Question 14 is 3 marks)**

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**15** Solve the simultaneous equations

$$\textcircled{1} \quad 2y^2 + x^2 = -6x + 42$$
$$\textcircled{2} \quad 2x + y = -3$$

Show clear algebraic working.

From ② $y = -3 - 2x$

$y^2 = (-3 - 2x)(-3 - 2x)$

$= 9 + 6x + 6x + 4x^2$

$= 4x^2 + 12x + 9$

sub in ①

$2(4x^2 + 12x + 9) + x^2 + 6x - 42 = 0$

$8x^2 + 24x + 18 + x^2 + 6x - 42 = 0$

$9x^2 + 30x - 24 = 0$

$\div 3$ $\quad 3x^2 + 10x - 8 = 0$

$(3x - 2)(x + 4) = 0$

$x = \frac{2}{3} \qquad x = -4$

using $y = -3 - 2x$

$y = -3 - 2 \times \frac{2}{3} \qquad y = -3 - 2 \times -4$

$= -\frac{13}{3} \qquad = 5$

$x = -4, y = 5 \qquad x = 2/3 \; y = -13/3$ ..............................

**(Total for Question 15 is 5 marks)**

**16** Use algebra to show that $0.3\dot{8}\dot{1} = \frac{21}{55}$

$100x = \cancel{38}^{37}.1818181...$

$x = 0.3818181...$

$99x = 37.8$

$x = \frac{37.8}{99}$

$\frac{37.8}{99} = \frac{378}{990}$

$\frac{378}{990} = \frac{21}{55}$

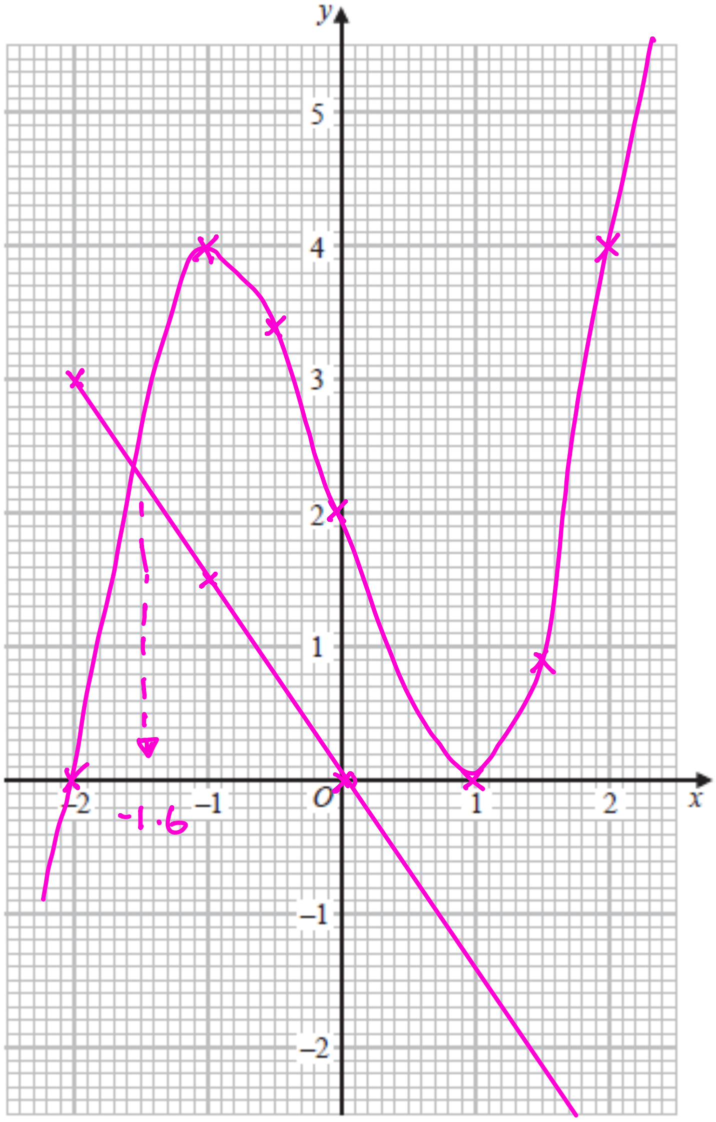
**(Total for Question 16 is 2 marks)**

**17** (*a*) Complete the table of values for $y = x^3 - 3x + 2$

| $x$ | $-2$ | $-1$ | $-0.5$ | $0$ | $1$ | $1.5$ | $2$ |
|---|---|---|---|---|---|---|---|
| $y$ | 0 | 4 | 3.4 | 2 | 0 | 0.9 | 4 |

**(2)**

(*b*) On the grid, draw the graph of $y = x^3 - 3x + 2$ for values of $x$ from $-2$ to 2




**(2)**

(*c*) By drawing a suitable straight line on the grid, use your graph to find an estimate for the solution of

$y = x^3 - 3x + 2$

$$2x^3 - 3x + 4 = 0$$

Give your answer correct to one decimal place.

$x = -1.6$

**(3)**

**(Total for Question 17 is 7 marks)**

**18** Prove algebraically that, for any three consecutive even numbers,

the sum of the squares of the smallest even number and the largest even number is 8 more than twice the square of the middle even number.

Let $n$ be any number

so $2n$ is always even

so $2n$ $2n+2$ and $2n+4$ are consecutive even numbers.

$(2n)^2 + (2n+4)^2$

$= 4n^2 + 4n^2 + 16n + 16$

$= 8n^2 + 16n + 16$

$(2n+2)^2$

$= 4n^2 + 8n + 4$

$2(2n+2)^2$

$= 8n^2 + 16n + 8$

$2(2n^2+2)^2 + 8$

$= 8n^2 + 16n + 16$

$8n^2 + 16n + 16 = 8n^2 + 16n + 16$

as required

**(Total for Question 18 is 3 marks)**

**19** Solve $\sqrt{3}(x - 2\sqrt{3}) = x + 2\sqrt{3}$

Give your answer in the form $a + b\sqrt{3}$ where $a$ and $b$ are integers.
Show your working clearly.

$\sqrt{3}x - 2\sqrt{3}\sqrt{3} = x + 2\sqrt{3}$

$\sqrt{3}x - 6 = x + 2\sqrt{3}$

$x\sqrt{3} - x = 6 + 2\sqrt{3}$

$x(\sqrt{3} - 1) = 6 + 2\sqrt{3}$

$x = \dfrac{6 + 2\sqrt{3}}{\sqrt{3} - 1}$

$= \dfrac{(6 + 2\sqrt{3})}{(\sqrt{3} - 1)} \times \dfrac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)}$

$= \dfrac{6\sqrt{3} + 6 + 6 + 2\sqrt{3}}{2} = \dfrac{8\sqrt{3} + 12}{2} = 4\sqrt{3} + 6$

$x =$ ......$6 + 4\sqrt{3}$......

**(Total for Question 19 is 4 marks)**

**20** Write

$$\frac{4x^2-17x-15}{2x-1}\times\frac{2x^2-7x+3}{x^2-25}+(29-4x)$$

as a single fraction in its simplest form.

$4x^2-17x-15 = 4x^2-20x+3x-15$

$4\times 15 = 60 \quad 3,20 \quad = 4x(x-5)+3(x-5)$

$= (4x+3)(x-5)$

$2x^2-7x+3 = 2x^2-6x-1x+3$

$2\times 3=6 \quad 1,6 \quad = 2x(x-3)-1(x-3)$

$=(2x-1)(x-3)$

$x^2-25 = (x+5)(x-5)$

$$\frac{(4x+3)\cancel{(x-5)}\times\cancel{(2x-1)}(x-3)}{\cancel{(2x-1)}(x+5)\cancel{(x-5)}}+29-4x$$

$$=\frac{(4x+3)(x-3)+(29-4x)(x+5)}{x+5}$$

$29 \times 5 = 145$

$$=\frac{\cancel{4x^2}-\cancel{12x}+\cancel{3x}-9+\cancel{29x}+145-\cancel{4x^2}-\cancel{20x}}{x+5}$$

$$=\frac{136}{x+5}$$

......................................................

**(Total for Question 20 is 4 marks)**

**21** $P$ is inversely proportional to $y^2$
When $y = 4$, $P = a$

($a$) Find a formula for $P$ in terms of $y$ and $a$

$P \propto \frac{1}{y^2}$ $\quad P = \frac{k}{y^2}$ $\quad a = \frac{k}{4^2}$ $\quad k = 16a$

so $P = \frac{16a}{y^2}$

$P = \frac{16a}{y^2}$

**(3)**

Given also that $y$ is directly proportional to $\sqrt{x}$ and when $x = a$, $P = 4a$

($b$) find a formula for $P$ in terms of $x$ and $a$

$y^2 = \frac{16a}{P}$

$= \frac{16a}{4a}$

$y = 2$

$y \propto \sqrt{x}$

$y = k\sqrt{x}$

$= k\sqrt{a}$

$2 = k\sqrt{a}$

$k = \frac{2}{\sqrt{a}}$ $\quad \therefore y = \frac{2}{\sqrt{a}}\sqrt{x}$

$P = 16a \div \left(\frac{2}{\sqrt{a}}\sqrt{x}\right)^2 = 16a \times \frac{a}{4x} = \frac{16a^2}{4x}$

$P = \frac{4a^2}{x}$

**(3)**

**(Total for Question 21 is 6 marks)**

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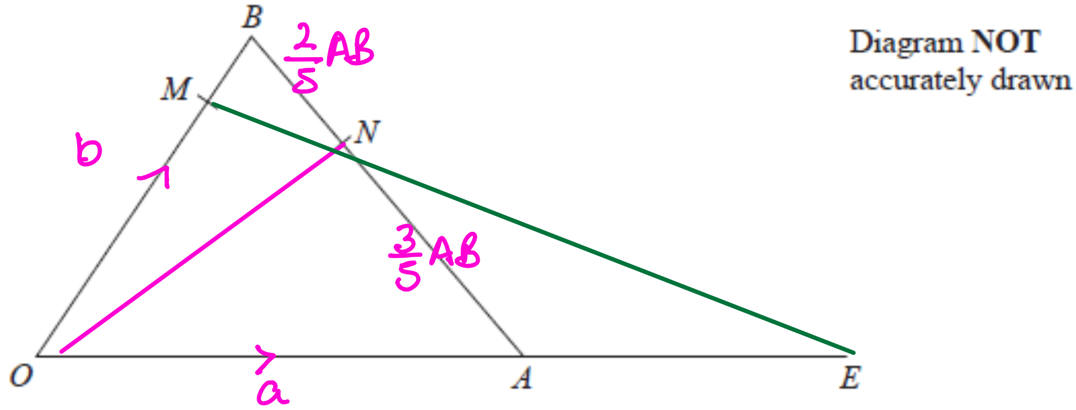
**22** The diagram shows triangle $OAB$ with $OA$ extended to $E$




$\overrightarrow{OA} = \mathbf{a}$ $\quad \overrightarrow{OB} = \mathbf{b}$

$\overrightarrow{OM} = \frac{4}{5}b$ $\quad \overrightarrow{MB} = \frac{1}{5}b$

$M$ is the point on $OB$ such that $OM : MB = 4 : 1$
$N$ is the point on $AB$ such that $AN : NB = 3 : 2$
$OA : AE = 5 : 3$

(*a*) Find an expression for $\overrightarrow{ON}$ in terms of **a** and **b**
Give your answer in its simplest form.

$\overrightarrow{BA} = a - b$

$\overrightarrow{ON} = \overrightarrow{OB} + BN$

$= b + \frac{2}{5}(a - b)$

$= \frac{2}{5}a + \frac{3}{5}b$

$= \frac{1}{5}(2a + 3b)$

$\overrightarrow{ON} = \frac{1}{5}(2a+3b)$

**(2)**

(*b*) Use a vector method to show that *MNE* is a straight line.

$\overrightarrow{OE} = \frac{8}{5}a$

$\overrightarrow{ME} = -\frac{4}{5}b + \frac{8}{5}a \quad = \frac{8}{5}a - \frac{4}{5}b$

$\overrightarrow{NE} = \frac{3}{5}(a-b) + \frac{3}{5}a$

$= \frac{6}{5}a - \frac{3}{5}b$

$\overrightarrow{MN} = \frac{1}{5}b + \frac{2}{5}(a-b) = \frac{2}{5}a - \frac{1}{5}b$

$\overrightarrow{ME} = 4\overrightarrow{MN}$

$\therefore$ MNE is a straight line

**(3)**

**(Total for Question 22 is 5 marks)**

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**TOTAL FOR PAPER IS 80 MARKS**